$$x = log_{\underset{\text{base}}{b}}\ \underset{\text{argomento}}{a}$$

Definizione:

[1] $$\boxed{x = log_b\, a \quad \Leftrightarrow \quad b^x = a}$$

Esempi:

$log_{10} 100 = 2 \quad \xrightarrow{\text{infatti}} \quad 10^2 = 100$

$log_2 8 = 3 \quad \xrightarrow{\text{infatti}} \quad 2^3 = 8$

$log_5 0 = 1 \quad \xrightarrow{\text{infatti}} \quad 5^0 = 1$

$log_b 0 = 1 \quad \xrightarrow{\text{infatti}} \quad b^0 = 1\ se\ b \neq 0$

Proprietà:

[2] $$\boxed{b^{log_b a} = a}$$

Dimostrazione:
dalla definizione [1]: $b^{log_b a} = b^x = a$

[3] $$\boxed{log_b(a \cdot c) = log_b\, a + log_b\, c}$$

Dimostrazione:

$$b^{log_b(a \cdot c)} = a \cdot c = b^{log_b a} \cdot b^{log_b c} = b^{log_b a + log_b c} \Rightarrow b^{log_b(a \cdot c)} = b^{log_b a + log_b c}$$

$$\Rightarrow log_b(a \cdot c) = log_b\, a + log_b\, c$$

[4] $$\boxed{log_b\, a^n = n \cdot log_b\, a}$$

Dimostrazione

$$log_b\, a^n = log_b \underbrace{(a \cdot a \dots \cdot a)}_{\text{n volte}} \overset{[3]}{\hat{=}} \underbrace{log_b\, a + \dots + log_b\, a}_{\text{n volte}} = n \cdot log_b\, a$$

Caso particolare:

[5] $$\boxed{log_b\, a^{-1} = log_b \frac{1}{a} = -\, log_b\, a}$$

[6] $$\boxed{log_b \frac{a}{c} = log_b\, a - log_b\, c}$$

Dimostrazione:

$$log_b \frac{a}{c} = log_b \left(a \cdot \frac{1}{c}\right) \overset{[3]}{\hat{=}} log_b\, a + log_b \frac{1}{c} \overset{[5]}{\hat{=}} log_b\, a - log_b\, c$$

Cambio di base:

[7] $$\boxed{log_b\, a = \frac{log_n\, a}{log_n\, b}}$$

Esempio (il log in base 2 non c'è sulla calcolatrice, il log in base 10 si ..):

$$log_2 8 = \frac{log_{10} 8}{log_{10} 2} \cong \frac{0.903}{0.301} = 3$$